Appendix B

Notation

This appendix contains a summary of the notation used in this book.

Math notation

Expression	Read as	Used to denote
a, b, x, y		variables
$=$	is equal to	expressions that have the same value
$\stackrel{\text{def}}{=}$	is defined as	new variable definitions
$a + b$	a plus b	the combined lengths of a and b
$a - b$	a minus b	the difference in lengths between a and b
$a \times b = ab$	a times b	the area of a rectangle
$a^2 \stackrel{\text{def}}{=} aa$	a squared	the area of a square of side length a
$a^3 \stackrel{\text{def}}{=} aaa$	a cubed	the volume of a cube of side length a
a^n	a exponent n	a multiplied by itself n times
$\sqrt{a} \stackrel{\text{def}}{=} a^{\frac{1}{2}}$	square root of a	the side length of a square of area a
$\sqrt[3]{a} \stackrel{\text{def}}{=} a^{\frac{1}{3}}$	cube root of a	the side length of a cube with volume a
$a/b = \frac{a}{b}$	a divided by b	a parts of a whole split into b parts
$a^{-1} \stackrel{\text{def}}{=} \frac{1}{a}$	one over a	division by a
$f(x)$	f of x	the function f applied to input x
f^{-1}	f inverse	the inverse function of $f(x)$
$f \circ g$	f compose g	function composition; $f \circ g(x) \stackrel{\text{def}}{=} f(g(x))$
e^x	e to the x	the exponential function base e
$\ln(x)$	natural log of x	the logarithm base e
a^x	a to the x	the exponential function base a
$\log_a(x)$	log base a of x	the logarithm base a
θ, ϕ	*theta*, *phi*	angles
sin, cos, tan	sin, cos, tan	trigonometric ratios
%	percent	proportions of a total; $a\% \stackrel{\text{def}}{=} \frac{a}{100}$

Set notation

You don't need a lot of fancy notation to do math, but it really helps if you know a little bit of set notation.

Symbol	Read as	Denotes
$\{\ldots\}$	the set ...	definition of a set
$\mid$	such that	describe or restrict the elements of a set
$\mathbb{N}$	the naturals	the set $\mathbb{N} \stackrel{\text{def}}{=} \{0,1,2,\ldots\}$. Also $\mathbb{N}_+ \stackrel{\text{def}}{=} \mathbb{N}\backslash\{0\}$.
$\mathbb{Z}$	the integers	the set $\mathbb{Z} \stackrel{\text{def}}{=} \{\ldots,-2,-1,0,1,2,3,\ldots\}$
$\mathbb{Q}$	the rationals	the set of fractions of integers
$\mathbb{R}$	the reals	the set of real numbers
$\mathbb{C}$		the set of complex numbers
$\mathbb{F}_q$	finite field	the set $\{0,1,2,3,\ldots,q-1\}$
$\subset$	subset	one set strictly contained in another
$\subseteq$	subset or equal	containment or equality
$\cup$	union	the combined elements from two sets
$\cap$	intersection	the elements two sets have in common
$S\backslash T$	S set minus T	the elements of S that are not in T
$a \in S$	a in S	a is an element of set S
$a \notin S$	a not in S	a is not an element of set S
$\forall x$	for all x	a statement that holds for all x
$\exists x$	there exists x	an existence statement
$\nexists x$	there doesn't exist x	a non-existence statement
$S \times T$	Cartesian product	all pairs (s,t) where $s \in S$ and $t \in T$

An example of a condensed math statement that uses set notation is "$\nexists m, n \in \mathbb{Z}$ such that $\frac{m}{n} = \sqrt{2}$," which reads "there don't exist integers m and n whose fraction equals $\sqrt{2}$." Since we identify the set of fractions of integers with the rationals, this statement is equivalent to the shorter "$\sqrt{2} \notin \mathbb{Q}$," which reads "$\sqrt{2}$ is irrational."

Vectors notation

Expression	Denotes
$\mathbb{R}^n$	the set of n-dimensional real vectors
$\vec{v}$	a vector
(v_x, v_y)	vector in component notation
$v_x\hat{\imath} + v_y\hat{\jmath}$	vector in unit vector notation
$\\|\vec{v}\\|\angle\theta$	vector in length-and-direction notation
$\\|\vec{v}\\|$	length of the vector $\vec{v}$
θ	angle the vector $\vec{v}$ makes with the x-axis
$\hat{v} \stackrel{\text{def}}{=} \frac{\vec{v}}{\\|\vec{v}\\|}$	unit vector in the same direction as $\vec{v}$
$\vec{u} \cdot \vec{v}$	dot product of the vectors $\vec{u}$ and $\vec{v}$
$\vec{u} \times \vec{v}$	cross product of the vectors $\vec{u}$ and $\vec{v}$

Complex numbers notation

Expression	Denotes
$\mathbb{C}$	the set of complex numbers $\mathbb{C} \stackrel{\text{def}}{=} \{a + bi \mid a, b \in \mathbb{R}\}$
i	the unit imaginary number $i \stackrel{\text{def}}{=} \sqrt{-1}$ and $i^2 = -1$
$\text{Re}\{z\} = a$	real part of $z = a + bi$
$\text{Im}\{z\} = b$	imaginary part of $z = a + bi$
$\lvert z\rvert\angle\varphi_z$	polar representation of $z = \lvert z\rvert \cos\varphi_z + i\lvert z\rvert \sin\varphi_z$
$\lvert z\rvert = \sqrt{a^2 + b^2}$	magnitude of $z = a + bi$
$\varphi_z = \tan^{-1}(b/a)$	phase or argument of $z = a + bi$
$\bar{z} = a - bi$	complex conjugate of $z = a + bi$
$\mathbb{C}^n$	the set of n-dimensional complex vectors

Vector space notation

Expression	Denotes
U, V, W	vector spaces
$W \subseteq V$	vector space W subspace of vector space V
$\{\vec{v} \in V \mid \langle \texttt{cond} \rangle\}$	subspace of vectors in V satisfying condition $\langle \texttt{cond} \rangle$
$\text{span}(\vec{v}_1, \ldots, \vec{v}_n)$	span of vectors $\vec{v}_1, \ldots, \vec{v}_n$
$\dim(U)$	dimension of vector space U
$\mathcal{R}(M)$	row space of M
$\mathcal{N}(M)$	null space of M
$\mathcal{C}(M)$	column space of M
$\mathcal{N}(M^{\mathsf{T}})$	left null space of M
$\text{rank}(M)$	rank of M; $\text{rank}(M) \stackrel{\text{def}}{=} \dim(\mathcal{R}(M)) = \dim(\mathcal{C}(M))$
$\text{nullity}(M)$	nullity of M; $\text{nullity}(M) \stackrel{\text{def}}{=} \dim(\mathcal{N}(M))$
B_s	the standard basis
$\{\vec{e}_1, \ldots, \vec{e}_n\}$	an orthogonal basis
$\{\hat{e}_1, \ldots, \hat{e}_n\}$	an orthonormal basis
${}_{B'}[\mathbb{1}]_B$	the change-of-basis matrix from basis B to basis B'
Π_S	projection onto subspace S
$\Pi_{S^\perp}$	projection onto the orthogonal complement of S

Abstract vector spaces notation

Expression	Denotes
$(V, F, +, \cdot)$	abstract vector space of vectors from the set V, whose coefficients are from the field F, addition operation "+" and scalar-multiplication operation "$\cdot$ "
$\mathbf{u}, \mathbf{v}, \mathbf{w}$	abstract vectors
$\langle \mathbf{u}, \mathbf{v} \rangle$	inner product of vectors $\mathbf{u}$ and $\mathbf{v}$
$\lVert \mathbf{u} \rVert$	norm of $\mathbf{u}$
$d(\mathbf{u}, \mathbf{v})$	distance between $\mathbf{u}$ and $\mathbf{v}$

Notation for matrices and matrix operations

Expression	Denotes
$\mathbb{R}^{m \times n}$	the set of $m \times n$ matrices with real entries
A	a matrix
a_{ij}	entry in the i^{th} row and j^{th} column of A
$\lvert A \rvert$	determinant of A, also denoted $\det(A)$
A^{-1}	matrix inverse
A^{T}	matrix transpose
$\mathbb{1}$	identity matrix; $\mathbb{1}A = A\mathbb{1} = A$ and $\mathbb{1}\vec{v} = \vec{v}$
AB	matrix-matrix product
$A\vec{v}$	matrix-vector product
$\vec{w}^{\mathsf{T}}A$	vector-matrix product
$\vec{u}^{\mathsf{T}}\vec{v}$	vector-vector inner product; $\vec{u}^{\mathsf{T}}\vec{v} \stackrel{\text{def}}{=} \vec{u} \cdot \vec{v}$
$\vec{u}\vec{v}^{\mathsf{T}}$	vector-vector outer product
$\text{ref}(A)$	row echelon form of A
$\text{rref}(A)$	reduced row echelon form of A
$\text{rank}(A)$	rank of $A \stackrel{\text{def}}{=}$ number of pivots in $\text{rref}(A)$
$A \sim A'$	matrix A' obtained from matrix A by row operations
$\mathcal{R}_1, \mathcal{R}_2, \ldots$	row operations, of which there are three types: $\rightarrow$ $R_i \leftarrow R_i + kR_j$: add k-times row j to row i $\rightarrow$ $R_i \leftrightarrow R_j$: swap rows i and j $\rightarrow$ $R_i \leftarrow mR_i$: multiply row i by constant m
$E_{\mathcal{R}}$	elementary matrix for row operation $\mathcal{R}$; $\mathcal{R}(M) \stackrel{\text{def}}{=} E_{\mathcal{R}}M$
$[\, A \mid \vec{b} \,]$	augmented matrix containing matrix A and vector $\vec{b}$
$[\, A \mid B \,]$	augmented matrix array containing matrices A and B
M_{ij}	minor associated with entry a_{ij}. See page 201.
$\text{adj}(A)$	adjugate matrix of A. See page 203.
$(A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}$	Moore–Penrose inverse of A. See page 414.
$\mathbb{C}^{m \times n}$	the set of $m \times n$ matrices with complex entries
$A^{\dagger}$	Hermitian transpose; $A^{\dagger} \stackrel{\text{def}}{=} (\overline{A})^{\mathsf{T}}$

Notation for linear transformations

Expression	Denotes
$T : \mathbb{R}^n \to \mathbb{R}^m$	linear transformation T from the input space $\mathbb{R}^n$ to the output space $\mathbb{R}^m$
$M_T \in \mathbb{R}^{m \times n}$	matrix representation of T
$\text{Im}(T) = \mathcal{C}(M_T)$	the image space of T = column space of M_T
$\text{Ker}(T) = \mathcal{N}(M_T)$	the kernel of T = null space of M_T
$S \circ T(\vec{x})$	composition of linear transformations; $S \circ T(\vec{x}) \stackrel{\text{def}}{=} S(T(\vec{x})) = M_S M_T \vec{x}$
$M \in \mathbb{R}^{m \times n}$	an $m \times n$ matrix
$T_M : \mathbb{R}^n \to \mathbb{R}^m$	the linear transformation defined as $T_M(\vec{v}) \stackrel{\text{def}}{=} M\vec{v}$
$T_{M^{\mathsf{T}}} : \mathbb{R}^m \to \mathbb{R}^n$	the adjoint linear transformation $T_{M^{\mathsf{T}}}(\vec{a}) \stackrel{\text{def}}{=} \vec{a}^{\mathsf{T}} M$

Matrix decompositions

Expression	Denotes
$A \in \mathbb{R}^{n \times n}$	a matrix (assume diagonalizable)
$p_A(\lambda) \stackrel{\text{def}}{=} \lvert A - \lambda \mathbb{1} \rvert$	characteristic polynomial of A
$\lambda_1, \ldots, \lambda_n$	eigenvalues of A = roots of $p_A(\lambda)$
$\Lambda \in \mathbb{R}^{n \times n}$	diagonal matrix of eigenvalues of A
$\vec{e}_{\lambda_1}, \ldots, \vec{e}_{\lambda_n}$	eigenvectors of A
$Q \in \mathbb{R}^{n \times n}$	matrix whose columns are eigenvectors of A
$A = Q\Lambda Q^{-1}$	eigendecomposition of A
$A = O\Lambda O^{\mathsf{T}}$	eigendecomposition of a normal matrix
$B \in \mathbb{R}^{m \times n}$	a generic matrix
$\sigma_1, \sigma_2, \ldots$	singular values of B
$\Sigma \in \mathbb{R}^{m \times n}$	matrix of singular values of B
$\vec{u}_1, \ldots, \vec{u}_m$	left singular vectors of B
$U \in \mathbb{R}^{m \times m}$	matrix of left singular vectors of B
$\vec{v}_1, \ldots, \vec{v}_n$	right singular vectors of B
$V \in \mathbb{R}^{n \times n}$	matrix of right singular vectors of B
$B = U\Sigma V^{\mathsf{T}}$	singular value decomposition of B